

Geometrical realization of low dimensional complete intersections*

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Abstract This paper aims to give some examples of diffeomorphic (or homeomorphic) low dimensional complete intersections, which can be considered as a geometrical realization of classification theorems about complete intersections. A conjecture of Libgober and Wood [9, §9](Topology. **21**, 1982, 469–482) will be confirmed by one of examples.

Keywords complete intersections, realization, classification

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1 Introduction

Let $X_n(\underline{d}) \subset \mathbb{C}P^{n+r}$ be a smooth complete intersection of multidegree $\underline{d} := (d_1, \dots, d_r)$, the product $d_1 d_2 \cdots d_r$ is called total degree, denoted by d . The celebrated Lefschetz hyperplane section theorem asserts that the pair $(\mathbb{C}P^{n+r}, X_n(\underline{d}))$ is n -connected. Thom showed that the complex dimension n and the multidegree \underline{d} determine the diffeomorphism type of $X_n(\underline{d})$. However, the multidegree is not a topological invariant. An interesting and challenging problem is to classify complete intersections by numerical topological invariants, such as total degree, the Pontrjagin classes and Euler characteristics, which usually are polynomials on the multidegree (see Section 2).

In dimension 1, the classification of complete intersections follows from the classical theory of Riemann surfaces. In dimension 3, appealing to general classification theorems in differential topology, the classification is established by Jupp [7] and Wall [12], while in dimension 2 the homeomorphism classification has been settled by Freedman [6]. Ebeling [3] and Libgober-Wood [10] independently found examples of homeomorphic complex 2-dimensional complete intersections but not diffeomorphic. In [4], Fang and Klaus proved that, in dimensions $n = 4$, two complete intersections $X_n(\underline{d})$ and $X_n(\underline{d}')$ are homeomorphic if and only if they have the same total degree, Pontrjagin classes and Euler characteristics. Furthermore, Fang and the first author [5] generalized this homeomorphism result to dimension $n = 5, 6, 7$.

Theorem 1.1 (Fang-Klaus & Fang-Wang). *Two complete intersections $X_n(\underline{d})$ and $X_n(\underline{d}')$ are homeomorphic if and only if they have the same total degree, Pontrjagin classes and Euler characteristics, provided $n = 4, 5, 6, 7$.*

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With the help of Kreck's modified surgery theory [8], Traving [11] obtained partial classification results in higher dimensions under some restrictions on the total degree. Particularly, to the prime factorization of total degree $d = \prod_{p \text{ primes}} p^{\nu_p(d)}$, under the assumption that $\nu_p(d) \geq \frac{2n+1}{2(p-1)} + 1$ for all primes p with $p(p-1) \leq n+1$, Traving proved the following result (see [8, Theorem A] or [11]).

Theorem 1.2 (Traving). *Two complete intersections $X_n(\underline{d})$ and $X_n(\underline{d}')$ of complex dimension $n > 2$ fulfilling the assumption above for the total degree are diffeomorphic if and only if the total degrees, the Pontrjagin classes and the Euler characteristics agree.*

It is well known all complete intersections of fixed multidegree are diffeomorphic. On the other hand, there exist diffeomorphic complete intersections with different multidegrees. For lower dimensions, such as complex dimensions 2, 3, 4, 5, the diffeomorphic examples can be found in [1, 2, 9, 13].

The aim of this paper is to give examples of diffeomorphic (or homeomorphic) complex n -dimensional complete intersections, mainly $n \leq 7$, which can be considered as a geometrical realization of above Theorems 1.1 and 1.2. On the other hand, Libgober and Wood [9, §9] conjectured that there exist the diffeomorphic complex 3-dimensional complete intersections with different first Chern classes, which give an example of a disconnected moduli space. In this paper, the above conjecture will be confirmed by one of the listed examples.

These examples are partially found by computer searching. Note that, according to [9, Proposition 7.3], if $X_n(d_1, \dots, d_r) \subset \mathbb{C}P^{n+r}$ satisfies $n > 2$ and $r \leq \frac{n+2}{2}$, then the total degree and Pontrjagin classes of $X_n(d_1, \dots, d_r)$ determine the multidegree. Thus, it is impossible to find out such an homeomorphic or diffeomorphic example in which the codimension r is small relative to complex dimension n . It is worth stating that the degrees and codimensions in our examples will be as smaller as possible.

2 Characteristic classes of complete intersections

For a complete intersection $X_n(\underline{d})$, let H be the restriction of the dual bundle of the canonical line bundle over $\mathbb{C}P^{n+r}$ to $X_n(\underline{d})$, and $x = c_1(H) \in H^2(X_n(\underline{d}); \mathbb{Z})$. Associate the multidegree $\underline{d} = (d_1, d_2, \dots, d_r)$, define the power sums $s_i = \sum_{j=1}^r d_j^i$ for $1 \leq i \leq n$. Then the Chern classes, Pontrjagin classes and Euler characteristic are presented as follows ([9]):

$$\begin{aligned} c_k &= \frac{1}{k!} g_k(n+r+1-s_1, \dots, n+r+1-s_k) x^k, 1 \leq k \leq n; \\ p_k &= \frac{1}{k!} g_k(n+r+1-s_2, \dots, n+r+1-s_{2k}) x^{2k}, 1 \leq k \leq \left\lfloor \frac{n}{2} \right\rfloor; \\ e &= c_n(X_n(\underline{d})) \cap [X_n(\underline{d})] = d \frac{1}{n!} g_n(n+r+1-s_1, \dots, n+r+1-s_n). \end{aligned}$$

Where $g_k(k \geq 1)$ are the polynomials that can be iteratively computed from the Newton formula:

$$s_k - g_1(s_1) s_{k-1} + \frac{1}{2} g_2(s_1, s_2) s_{k-2} + \dots + (-1)^k \frac{1}{k!} g_k(s_1, s_2, \dots, s_k) k = 0.$$

For example, here are the first seven g_k 's.

$$\begin{aligned}
g_1(s_1) &= s_1, \\
g_2(s_1, s_2) &= s_1^2 - s_2, \\
g_3(s_1, s_2, s_3) &= s_1^3 - 3s_1s_2 + 2s_3, \\
g_4(s_1, \dots, s_4) &= s_1^4 - 6s_1^2s_2 + 8s_1s_3 + 3s_2^2 - 6s_4, \\
g_5(s_1, \dots, s_5) &= s_1^5 - 10s_1^3s_2 + 20s_1^2s_3 - 30s_1s_4 + 15s_1s_2^2 - 20s_2s_3 + 24s_5, \\
g_6(s_1, \dots, s_6) &= s_1^6 - 15s_1^4s_2 + 40s_1^3s_3 - 90s_1^2s_4 + 45s_1^2s_2^2 - 120s_1s_2s_3 \\
&\quad + 144s_1s_5 - 15s_2^3 + 90s_2s_4 + 40s_3^2 - 120s_6, \\
g_7(s_1, \dots, s_7) &= s_1^7 - 21s_1^5s_2 + 70s_1^4s_3 - 210s_1^3s_4 + 105s_1^3s_2^2 - 420s_1^2s_2s_3 \\
&\quad + 504s_1^2s_5 - 105s_1s_2^3 + 630s_1s_2s_4 + 280s_1s_3^2 - 840s_1s_6 \\
&\quad + 210s_2^2s_3 - 504s_2s_5 - 420s_3s_4 + 720s_7.
\end{aligned}$$

Note that the k^{th} Chern classes c_k and Pontrjagin class p_k are integral multiples of x^k and x^{2k} , so we can compare this invariant for different complete intersections. For convenience, throughout the rest of the paper, the Chern class c_k and Pontrjagin class p_k of $X_n(\underline{d})$ are viewed as the multiple of x^k and x^{2k} respectively.

3 Examples of diffeomorphic (homeomorphic) low dimensional complete intersections

In this section, some examples of diffeomorphic or homeomorphic complex n -dimensional complete intersections will be listed for $n = 2, 3, \dots, 7$, and one of examples will confirm the conjecture of Libgober and Wood [9, §9].

3.1 Complex 2-dimensional complete intersections

For complex 2-dimensional complete intersection $X_2(d_1, \dots, d_r)$, the total degree, first Chern class, Pontrjagin class and Euler characteristic are as follows:

$$\begin{aligned}
d &= d_1 \times \dots \times d_r, \\
c_1 &= 3 + r - s_1, \\
p_1 &= 3 + r - s_2, \\
e &= \frac{d}{2} [(3 + r - s_1)^2 - (3 + r - s_2)].
\end{aligned}$$

Two complete intersections $X_2(d_1, \dots, d_r)$ and $X_2(d'_1, \dots, d'_s)$, which satisfy the following conditions,

$$\begin{aligned}
d \cdot p_1 &= d' \cdot p'_1, \\
e &= e', \\
c_1 &\equiv c'_1 \pmod{2}, \\
c_1 &\neq c'_1.
\end{aligned}$$

will give an example of homeomorphic but non-diffeomorphic complex 2-dimensional complete intersections (see [3, 10]). Note that, Ebeling [3] and Libgober-Wood [10] independently found that $X_2(10, 7, 7, 6, 6, 3, 3)$ and $X_2(9, 5, 3, 3, 3, 3, 2, 2)$ are homeomorphic but not diffeomorphic. Here, some other examples with lower codimensions are listed.

Example 3.1. The following invariants give seven examples of pairs of complex 2-dimensional complete intersections which are homeomorphic but not diffeomorphic. The two smoothings in each pair have distinct c_1 's.

$X_2(\underline{d})$	$d \cdot p_1$	e	c_1
$X_2(6, 5, 3)$	-5760	5760	-8
$X_2(5, 2, 2, 2, 2, 2)$	-5760	5760	-6
$X_2(14, 14, 8)$	-705600	1058400	-30
$X_2(14, 7, 5, 5)$	-705600	1058400	-24
$X_2(15, 11, 10, 3)$	-2217600	3643200	-32
$X_2(11, 10, 5, 2, 2, 2, 2)$	-2217600	3643200	-24
$X_2(10, 9, 7, 7, 7)$	-9878400	20744640	-32
$X_2(7, 7, 7, 5, 2, 2, 2, 2)$	-9878400	20744640	-24
$X_2(10, 9, 9, 6, 2)$	-2857680	5239080	-28
$X_2(10, 7, 7, 3, 3, 3)$	-2857680	5239080	-24
$X_2(15, 13, 7, 3, 2)$	-3669120	6027840	-32
$X_2(13, 7, 5, 2, 2, 2, 2)$	-3669120	6027840	-24
$X_2(10, 7, 7, 6, 3, 3)$	-6429780	12859560	-27
$X_2(9, 5, 3, 3, 3, 3, 3, 2, 2)$	-6429780	12859560	-21

Table 1: Homeomorphic but non-diffeomorphic 2-dim complete intersections

3.2 Complex 3-dimensional complete intersections

For complex 3-dimensional complete intersection $X_3(d_1, \dots, d_r)$, the related topological characteristic classes are as follows:

$$\begin{aligned}
d &= d_1 \times \dots \times d_r, \\
p_1 &= 4 + r - s_2, \\
e &= \frac{d}{6} [(4 + r - s_1)^3 - 3(4 + r - s_1)(4 + r - s_2) + 2(4 + r - s_3)], \\
c_1 &= 4 + r - s_1.
\end{aligned}$$

Two complex 3-dimensional complete intersections are diffeomorphic, if and only if they have the same total degree d , the first Pontrjagin class p_1 and Euler characteristics ([7, 12]). In [9, §9], Libgober and Wood conjectured that there exist such examples of diffeomorphic complex 3-dimensional complete intersections with different first Chern classes, which would give an example of a disconnected moduli space. Brückmann [1] show that there exist the diffeomorphic complex 3-dimensional complete intersections belonging to components of the moduli space of different dimensions, but with the same first Chern classes. In the following, some examples of diffeomorphic complex 3-dimensional complete intersections will be listed, and the conjecture in [9] is confirmed by Table 3.

Example 3.2. The following invariants give some pairs of diffeomorphic complex 3-dimensional complete intersections, where the ones with different first Chern classes c_1 give an example of a disconnected moduli space.

$X_3(\underline{d})$	d	p_1	e	c_1
$X_3(20, 20, 11, 7, 4)$	123200	-977	-6974721600	-53
$X_3(22, 16, 14, 5, 5)$	123200	-977	-6974721600	-53
$X_3(14, 14, 5, 4, 4, 4)$	62720	-455	-1068748800	-35
$X_3(16, 10, 7, 7, 2, 2, 2)$	62720	-455	-1068748800	-35

Table 2: Diffeomorphic 3-dim complete intersections with same c_1

$X_3(\underline{d})$	d	p_1	e	c_1
$X_3(70, 16, 16, 14, 7, 6)$	10536960	-5683	-7767425433600	-119
$X_3(56, 49, 8, 6, 5, 4, 4)$	10536960	-5683	-7767425433600	-121
$X_3(88, 28, 19, 14, 6, 6)$	23595264	-9147	-35445749391360	-151
$X_3(76, 56, 11, 7, 6, 6, 2)$	23595264	-9147	-35445749391360	-153
$X_3(84, 29, 25, 25, 18, 7)$	191835000	-9510	-384536710530000	-178
$X_3(60, 58, 49, 9, 5, 5, 5)$	191835000	-9510	-384536710530000	-180

Table 3: Diffeomorphic 3-dim complete intersections with different c_1

3.3 Complex 4-dimensional complete intersections

For complex 4-dimensional complete intersection $X_4(d_1, \dots, d_r)$, we have the following invariants:

$$\begin{aligned}
d &= d_1 \times \dots \times d_r, \\
p_1 &= 5 + r - s_2, \\
p_2 &= \frac{1}{2} [(5 + r - s_2)^2 - (5 + r - s_4)], \\
e &= \frac{d}{24} [(5 + r - s_1)^4 - 6(5 + r - s_1)^2(5 + r - s_2) + 8(5 + r - s_1)(5 + r - s_3) \\
&\quad + 3(5 + r - s_2)^2 - 6(5 + r - s_4)]
\end{aligned}$$

According to Theorem 1.1, to construct homeomorphic complex 4-dimensional complete intersections, we need to find out two different multidegrees to satisfy the above invariants all agree. Furthermore, by Theorem 1.2, if $\nu_2(d) > 5.5$, then the homeomorphic 4-dimensional complete intersections are diffeomorphic.

Example 3.3. Table 4 give two pairs of homeomorphic complex 4-dimensional complete intersections.

Example 3.4. Table 5 give two pairs of diffeomorphic complex 4-dimensional complete intersections, since the total degree satisfy

$$\begin{aligned}
488980800 &= 2^6 \times 3^4 \times 5^2 \times 7^3 \times 11, \\
3953664000 &= 2^{13} \times 3^3 \times 5^3 \times 11 \times 13.
\end{aligned}$$

$X_4(\underline{d})$	d	p_1	p_2	e
$X_4(66, 63, 29, 23, 6, 4)$	66561264	-9736	65253028	11837353833553248
$X_4(69, 58, 36, 14, 11, 3)$	66561264	-9736	65253028	11837353833553248
$X_4(46, 44, 33, 27, 17, 15, 10)$	4598629200	-6472	25986916	546159737882484000
$X_4(45, 45, 34, 23, 22, 12, 11)$	4598629200	-6472	25986916	546159737882484000

Table 4: Homeomorphic 4-dim complete intersections

$X_4(\underline{d})$	d	p_1	p_2	e
$X_4(36, 33, 30, 20, 14, 7, 7)$	488980800	-3967	9807916	19704249035856000
$X_4(35, 35, 28, 22, 12, 9, 6)$	488980800	-3967	9807916	19704249035856000
$X_4(52, 44, 36, 25, 20, 12, 8)$	3953664000	-7157	32268711	546278189783040000
$X_4(50, 48, 32, 26, 22, 10, 9)$	3953664000	-7157	32268711	546278189783040000

Table 5: Diffeomorphic 4-dim complete intersections

3.4 Complex 5-dimensional complete intersections

For complex 5-dimensional complete intersection $X_5(d_1, \dots, d_r)$, we have the following invariants:

$$\begin{aligned}
d &= d_1 \times \dots \times d_r, \\
p_1 &= 6 + r - s_2, \\
p_2 &= \frac{1}{2}[(6 + r - s_2)^2 - (6 + r - s_4)], \\
e &= \frac{1}{5!}d[(6 + r - s_1)^5 - 10(6 + r - s_1)^3(6 + r - s_2) + 20(6 + r - s_1)^2(6 + r - s_3) \\
&\quad - 30(6 + r - s_1)(6 + r - s_4) + 15(6 + r - s_1)(6 + r - s_2)^2 \\
&\quad - 20(6 + r - s_2)(6 + r - s_3) + 24(6 + r - s_5)],
\end{aligned}$$

Two complex 5-dimensional complete intersections with the same d, p_1, p_2, e are homeomorphic. Furthermore, by Theorem 1.2, if $\nu_2(d) \geq 6.5$ and $\nu_3(d) \geq 3.75$, then they will be diffeomorphic.

Example 3.5. Consider the following Table 6, the two pairs of multidegrees have the same

\underline{d}	s_1	s_2	s_3	s_4	s_5	d
(112, 93, 91, 50, 45, 20, 18)	429	34723	3192813	311347699	31322739669	767763360000
(108, 105, 78, 62, 35, 25, 16)	429	34723	3192813	311347699	31322739669	767763360000
(54, 48, 30, 30, 13, 11, 11, 4)	201	7447	326979	15489571	763263411	14677977600
(55, 44, 39, 18, 18, 16, 6, 5)	201	7447	326979	15489571	763263411	14677977600

Table 6: Diffeomorphic 5-dim complete intersections

power sums s_1, \dots, s_5 and total degree d , then it is easily deduced that the corresponding

complex 5-dimensional complete intersections have the same Pontrjagin classes and Euler characteristic. Since total degrees have the following prime factorization,

$$\begin{aligned} 767763360000 &= 2^8 \times 3^5 \times 5^4 \times 7^2 \times 13 \times 31, \\ 14677977600 &= 2^9 \times 3^6 \times 5^2 \times 11^2 \times 13, \end{aligned}$$

the corresponding two pairs of complete intersections are diffeomorphic.

Example 3.6. The following table gives a diffeomorphic complete intersections with distinct codimensions, where the diffeomorphism comes from the prime factorization of total degree: $d = 104626080000 = 2^8 \times 3^7 \times 5^4 \times 13 \times 23$.

$X_5(\underline{d})$ (codim=9, 10)	d	p_1	p_2	e/d
$X_5(52, 50, 30, 27, 23, 18, 6, 5, 4)$	104626080000	-7748	37660770	-12876778992
$X_5(54, 46, 36, 25, 20, 15, 13, 3, 2, 2)$	104626080000	-7748	37660770	-12876778992

Table 7: Diffeomorphic 5-dim complete intersections with distinct codimensions

3.5 Complex 6-dimensional complete intersections

For complex 6-dimensional complete intersection $X_6(d_1, \dots, d_r)$, the related topological characteristic classes are as follows:

$$\begin{aligned} d &= d_1 \times \dots \times d_r, \\ p_1 &= 7 + r - s_2, \\ p_2 &= \frac{1}{2} [(7 + r - s_2)^2 - (7 + r - s_4)], \\ p_3 &= \frac{1}{6} [(7 + r - s_1)^3 - 3(7 + r - s_1)(7 + r - s_2) + 2(7 + r - s_3)], \\ e &= \frac{d}{6!} [(7 + r - s_1)^6 - 15(7 + r - s_1)^4(7 + r - s_2) + 40(7 + r - s_1)^3(7 + r - s_3) \\ &\quad - 90(7 + r - s_1)^2(7 + r - s_4) + 45(7 + r - s_1)^2(7 + r - s_2)^2 \\ &\quad - 120(7 + r - s_1)(7 + r - s_2)(7 + r - s_3) + 144(7 + r - s_1)(7 + r - s_5) \\ &\quad - 15(7 + r - s_2)^3 + 90(7 + r - s_2)(7 + r - s_4) \\ &\quad + 40(7 + r - s_3)^2 - 120(7 + r - s_6)], \end{aligned}$$

Two complex 6-dimensional complete intersections with the same d, p_1, p_2, p_3, e are homeomorphic. Furthermore, by Theorem 1.2, if $\nu_2(d) \geq 7.5$ and $\nu_3(d) \geq 4.25$, then they will be diffeomorphic.

Example 3.7. Consider the following two multidegrees ¹

$$\begin{aligned} \underline{d} &= (116, 114, 96, 78, 59, 55, 50, 40, 32, 22, 13, 9), \\ \underline{d}' &= (118, 110, 100, 72, 64, 57, 48, 39, 29, 26, 11, 10), \end{aligned}$$

¹The multidegrees in Examples 3.7 and 3.9 are introduced to the author by Guo Xianqiang on mathematical BBS: <http://bbs.emath.ac.cn/thread-5853-1-1.html>.

They have the same total degree d , powers sums s_1, \dots, s_7 .

$$\begin{aligned} d &= 52\,933\,656\,400\,035\,840\,000, \\ s_1 &= 684, \\ s_2 &= 54\,116, \\ s_3 &= 5\,008\,824, \\ s_4 &= 503\,305\,604, \\ s_5 &= 52\,970\,710\,824, \\ s_6 &= 5\,730\,100\,991\,396, \\ s_7 &= 630\,552\,267\,588\,024. \end{aligned}$$

Then the corresponding two complete intersections have the same total degree, Pontrjagin classes and Euler characteristic, so they are homeomorphic. Since the total degree has the prime factorization $d = 2^{19} \times 3^5 \times 5^4 \times 11^2 \times 13^2 \times 19 \times 29 \times 59$, the corresponding two complete intersections

$$\begin{aligned} X_6(116, 114, 96, 78, 59, 55, 50, 40, 32, 22, 13, 9), \\ X_6(118, 110, 100, 72, 64, 57, 48, 39, 29, 26, 11, 10), \end{aligned}$$

are diffeomorphic.

3.6 Complex 7-dimensional complete intersections

For complex 7-dimensional complete intersection $X_7(d_1, \dots, d_r)$, the related topological characteristic classes are as follows:

$$\begin{aligned} p_1 &= 8 + r - s_2, \\ p_2 &= \frac{1}{2}[(8 + r - s_2)^2 - (8 + r - s_4)], \\ p_3 &= \frac{1}{6}[(8 + r - s_1)^3 - 3(7 + r - s_1)(8 + r - s_2) + 2(8 + r - s_3)], \\ e &= \frac{d}{7!}[(8 + r - s_1)^7 - 21(8 + r - s_1)^5(8 + r - s_2) + 70(8 + r - s_1)^4(8 + r - s_3) \\ &\quad - 210(8 + r - s_1)^3(8 + r - s_4) + 105(8 + r - s_1)^3(8 + r - s_2)^2 \\ &\quad - 420(8 + r - s_1)^2(8 + r - s_2)(8 + r - s_3) + 504(8 + r - s_1)^2(8 + r - s_5) \\ &\quad - 105(8 + r - s_1)(8 + r - s_2)^3 + 630(8 + r - s_1)(8 + r - s_2)(8 + r - s_4) \\ &\quad + 280(8 + r - s_1)(8 + r - s_3)^2 - 840(8 + r - s_1)(8 + r - s_6) \\ &\quad + 210(8 + r - s_2)^2(8 + r - s_3) - 504(8 + r - s_2)(8 + r - s_5) \\ &\quad - 420(8 + r - s_3)(8 + r - s_4) + 720(8 + r - s_7)], \end{aligned}$$

Two complex 7-dimensional complete intersections with the same d, p_1, p_2, p_3, e are homeomorphic. Furthermore, by Theorem 1.2, if $\nu_2(d) \geq 8.5$ and $\nu_3(d) \geq 4.75$, then they will be diffeomorphic.

Example 3.8. Consider the multidegrees in Example 3.7, it is evidently that the corresponding two 7-dimensional complete intersections have the same total degree, Pontrjagin classes and Euler characteristic. Thus complete intersections

$$\begin{aligned} X_7(116, 114, 96, 78, 59, 55, 50, 40, 32, 22, 13, 9), \\ X_7(118, 110, 100, 72, 64, 57, 48, 39, 29, 26, 11, 10), \end{aligned}$$

are diffeomorphic by Theorem 1.2. □

Example 3.9. Consider the following two multidegrees

$$\begin{aligned} (596, 592, 556, 520, 480, 450, 438, 423, 408, 404, 381, 369, 327, 312, 300) \\ (600, 584, 564, 508, 492, 447, 436, 417, 416, 400, 390, 360, 333, 306, 303) \end{aligned}$$

They have the same total degree d , powers sums s_1, \dots, s_7 .

$$\begin{aligned} d &= 2\,895\,548\,222\,951\,602\,337\,839\,765\,820\,276\,736\,000\,000, \\ s_1 &= 6\,556, \\ s_2 &= 2\,994\,164, \\ s_3 &= 1\,424\,846\,116, \\ s_4 &= 703\,565\,690\,996, \\ s_5 &= 358\,728\,296\,599\,276, \\ s_6 &= 187\,921\,032\,698\,324\,444, \\ s_7 &= 100\,667\,444\,609\,447\,734\,036. \end{aligned}$$

Furthermore $\nu_2(d) = 28$ and $\nu_2(d) = 13$, so the corresponding two complete intersections

$$\begin{aligned} X_7(596, 592, 556, 520, 480, 450, 438, 423, 408, 404, 381, 369, 327, 312, 300), \\ X_7(600, 584, 564, 508, 492, 447, 436, 417, 416, 400, 390, 360, 333, 306, 303) \end{aligned}$$

are diffeomorphic. □

Example 3.10. For the following two multidegrees ²

$$\begin{aligned} \underline{d} &= (12, 16, 22, 26, 28, 45, 58, 59, 65, 69, 81, 85, 86, 91, 105, 106, \\ &\quad 108, 128, 132, 134, 144, 156, 168, 192, 200, 214, 242, 250, 272, 274), \\ \underline{d}' &= (13, 14, 24, 25, 29, 43, 54, 64, 66, 72, 78, 84, 88, 90, 96, 107, 121, \\ &\quad 125, 130, 136, 137, 162, 170, 182, 210, 212, 236, 256, 268, 276), \end{aligned}$$

²These two multidegrees were firstly appeared in <http://www.emath.ac.cn/florilegium/r011n30.htm> by Guo Xianqiang.

They have the same total degree d , powers sums s_1, \dots, s_{11} .

$$\begin{aligned}
d &= 43\,548\,968\,602\,421\,704\,369\,217\,485\,857\,781\,603\,627\,739\,564\,212\,224\,000\,000\,000, \\
s_1 &= 3\,568, \\
s_2 &= 601\,432, \\
s_3 &= 120\,991\,744, \\
s_4 &= 26\,887\,338\,904, \\
s_5 &= 6\,339\,294\,665\,608, \\
s_6 &= 1\,550\,802\,794\,333\,392, \\
s_7 &= 388\,678\,376\,991\,878\,944, \\
s_8 &= 99\,057\,950\,894\,851\,518\,184, \\
s_9 &= 25\,552\,911\,575\,591\,712\,680\,248, \\
s_{10} &= 6\,651\,777\,099\,183\,876\,642\,569\,152, \\
s_{11} &= 1\,743\,797\,004\,813\,063\,555\,915\,251\,344.
\end{aligned}$$

Then the corresponding two complete intersections are homeomorphic. Since $\nu_2(d) = 51$ and $\nu_2(d) = 18$, the corresponding two complete intersections

$$\begin{aligned}
&X_7(12, 16, 22, 26, 28, 45, 58, 59, 65, 69, 81, 85, 86, 91, 105, 106, \\
&\quad 108, 128, 132, 134, 144, 156, 168, 192, 200, 214, 242, 250, 272, 274), \\
&X_7(13, 14, 24, 25, 29, 43, 54, 64, 66, 72, 78, 84, 88, 90, 96, 107, 121, \\
&\quad 125, 130, 136, 137, 162, 170, 182, 210, 212, 236, 256, 268, 276),
\end{aligned}$$

are diffeomorphic. □

Remark 3.11. Note that, just like Examples 3.7 and 3.8, for the multidegrees appeared in Subsection 3.6, the corresponding complex 6-dimensional complete intersections are also diffeomorphic.

Remark 3.12. For higher dimensional complete intersections, e.g. $n = 8, 9, 10, 11$, multidegrees in Example 3.10 are still valid to satisfy that the n -dimensional complete intersections have the same total degree, Pontrjagin classes and Euler characteristic. However, because of the unknown classifications of complete intersections for higher dimensions, we can not give more examples of diffeomorphic or homeomorphic complete intersections.

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